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ABSTRACT

Joreskog's general method for analysis of covariance structures was developed for estimating a model involving structures of a very general form on means, variances, and covariances of multivariate observations. This method achieves a great deal of generality and flexibility, in that it is capable of handling most standard statistical models as well as many nonstandard and complicated ones. This paper describes a computer program for the method. When the variance-covariance matrix of the observed variables is unconstrained, the method may be used to estimate location parameters and to test linear hypotheses about them. For example, the program may be used to handle such standard problems as multivariate regression, ANOVA, and MANOVA. A unique feature of the method is that it can also be used when the variance-covariance matrix is constrained tc be of a certain form. Various other models involving correlated errors or errors of measurement can also be handled. An illustration of how input data is set up, and what the printout looks like for two small sets of data, are included. (Author/CK)

RESEARCH BULLETIN

RB-71-1

ACOVSM:

A GENERAL COMPUTER PROGRAM FOR ANALYSIS OF COVARIANCE STRUCTURES INCLUDING GENERALIZED MANOVA

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ACOVSM

A General Computer Program for Analysis of Covariance Structures Including Generalized MANOVA[1]

1. Introduction

In a previous paper, Jöreskog (1970a) developed a method for estimating a model involving structures of a very general form on means, variances and covariances of multivariate observations. With this method, a great deal of generality and flexibility is achieved in that the method is capable of handling most standard statistical models as well as many nonstandard and complicated ones. The purpose of this paper is to describe a computer program for this method.

When the variance-covariance matrix of the observed variables is unconstrained, the method may be used to estimate location parameters and to test linear hypotheses about these. For example, the program may be used to handle such standard problems as multivariate regression, ANOVA and MANOVA, although there may not be any advantage in using this particular program as compared to other existing programs. It can also be used for generalized MANOVA in the sense of Potthoff and Roy (1964), Khatri (1966) and Grizzle and Allen (1969) (see also Rao, 1959, 1965, 1966, 1967; Gleser and Olkin, 1966). A unique feature is that the method can be used also when the variance-covariance matrix is constrained to be of a certain form. In this case one can estimate the covariance structure as well as location parameters and, in large samples, one can test various hypotheses

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about the structure of the variance-covariance matrix. This is useful in many areas and problems particularly in the behavioral sciences. For example, one can handle such problems as analysis of congeneric tests, factor analysis, analysis of multitrait-multimethod data, analysis of simplexes and circumplexes, analysis of multitest-multioccasion data and growth data in general, analysis of mixed and random effects ANOVA and MANOVA, path analysis and linear structural equations (Jöreskog, 1970a-b, 1971). Various other models involving correlated errors or errors of measurement can also be handled.

1.1 The General Model

The general model considers a data matrix $X(N \times p)$ of N observations on p variates and assumes that the rows of X are independently distributed, each having a multivariate normal distribution with the same variance-covariance matrix Σ. It is assumed that

$$\mathcal{E}(X) = A\Xi P \quad , \tag{1}$$

where $A(N \times g) = (a_{\alpha s})$ and $P(h \times p) = (p_{ti})$ are known matrices of ranks g and h, respectively, $g \leq N$, $h \leq p$ and $\Xi(g \times h) = (\xi_{st})$ is a matrix of parameters; and that Σ has the form

$$\Sigma = B(\Lambda\Phi\Lambda' + \psi^2)B' + \Theta^2 \quad , \tag{2}$$

where the matrices $B(p \times q) = (\beta_{ik})$, $\Lambda(q \times r) = (\lambda_{km})$, the symmetric matrix $\Phi(r \times r) = (\phi_{mn})$ and the diagonal matrices $\psi(q \times q) = (\delta_{kl}\psi_k)$ and $\Theta(p \times p) = (\delta_{ij}\theta_i)$ are parameter matrices.

Thus the general model is one where means, variances and covariances are structured in terms of other sets of parameters that are to be estimated. In any application of this model, p, N and X will be given by the data, and g, h, q, r, A and P will be given by the particular application. In any such application we shall allow for any one of the parameters in Ξ, B, Λ, Φ, ψ and Θ to be known a priori and for one or more subsets of the remaining parameters to have identical but unknown values. Thus parameters are of three kinds: (i) fixed parameters that have been assigned given values, (ii) constrained parameters that are unknown but equal to one or more other parameters and (iii) free parameters that are unknown and not constrained to be equal to any other parameter.

The computer program estimates the free and constrained parameters of any such model by the maximum likelihood method and provides a test of goodness of fit of the whole model against the general alternative that P is square and Ξ and Σ are unconstrained. A test of a specified model (hypothesis) may be obtained, in large samples, by computing the maximum likelihood solution under the two models and then setting up the likelihood ratio test (see 1.5). In the special case when both Ξ and Σ are unconstrained, one may test a sequence of hypotheses of the form

$$C\Xi D = 0 \quad , \qquad (3)$$

where $C(s \times g)$ and $D(h \times t)$ are given matrices of ranks s and t, respectively.

1.2 Identification of Parameters

Before an attempt is made to estimate a model of this kind the identification problem must be examined. The identification problem depends on the specification of fixed, free and constrained parameters.

It should be noted that if B is replaced by BT_1^{-1}, Λ by $T_1 \Lambda T_2^{-1}$, Φ by $T_2 \Phi T_2'$ and ψ^2 by $T_1 \psi^2 T_1'$ while Θ is left unchanged, then Σ is unaffected. This holds for all nonsingular matrices $T_1(q \times q)$ and $T_2(r \times r)$ such that $T_1 \psi^2 T_1'$ is diagonal. Hence in order to obtain a unique set of parameters and a corresponding unique set of estimates, some restrictions must be imposed. In what follows it is assumed that all such indeterminacies have been eliminated by the specification of fixed and constrained parameters. To make sure that all indeterminacies have been eliminated, one should verify that the only transformations T_1 and T_2 that preserve the specifications about fixed and constrained parameters are identity matrices.

1.3 Matrices U, V and W

Since N may be large, the matrices X and A are not stored in the computer. Instead the information provided by these matrices is summarized in three matrices U, V and W defined as follows:

$$U(g \times g) = (1/N)A'A \tag{4}$$

$$V(g \times p) = (1/N)A'X \tag{5}$$

$$W(p \times p) = (1/N)X'X \quad . \tag{6}$$

1.4 Standard Case

It is convenient to distinguish between two different cases as follows:

Standard Case: Both Ξ and Σ are unconstrained.

Nonstandard Case: Otherwise.

In the standard case, the maximum likelihood estimates of Ξ and Σ are

$$\hat{\Xi} = U^{-1}VS^{-1}P'(PS^{-1}P')^{-1} \tag{7}$$

$$\hat{\Sigma} = S + Q'UQ \quad , \tag{8}$$

where

$$S = W - V'U^{-1}V \tag{9}$$

and

$$Q = U^{-1}V - \hat{\Xi}P \quad . \tag{10}$$

To test the hypothesis $C\Xi D = 0$ against $C\Xi D \neq 0$ one uses

$$S_e = D'(PS^{-1}P')^{-1}D \quad , \tag{11}$$

$$S_h = (C\hat{\Xi}D)'(CRC')^{-1}(C\hat{\Xi}D) \quad , \tag{12}$$

where

$$R = U^{-1} + QS^{-1}V'U^{-1} \quad . \tag{13}$$

Let the eigenvalues of $S_hS_e^{-1}$ be $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_t$. The program gives the three test statistics

$$\text{Largest Root} = \lambda_1$$

$$\text{Sum of Roots} = \sum_{i=1}^{t} \lambda_i$$

$$\text{Likelihood Ratio} = 1/\prod_{i=1}^{t} (1 + \lambda_i) \quad .$$

The largest root test, due to Roy (1953), can be used with Heck's (1960) tables. The sum of roots test is due to Lawley (1938) and Hotelling (1951). The likelihood ratio test is an extension of Wilks' (1932) λ -test and can be used with correction tables provided by Schatzoff (1966). When N is large, $-[N - g - (p - h) - \frac{1}{2}(t - s + 1)]$ times the likelihood ratio is approximately distributed as χ^2 with st degrees of freedom.

It should be noted that if P is square and nonsingular, formulas (7), (8), (10) and (12) reduce to the ordinary formulas for MANOVA, i.e.,

$$\hat{\Xi} = U^{-1}VP^{-1} \tag{7a}$$

$$\hat{\Sigma} = S \tag{8a}$$

$$Q = 0 \tag{10a}$$

$$S_h = (C\hat{\Xi}D)'(CU^{-1}C')^{-1}(C\hat{\Xi}D) \quad . \tag{12a}$$

1.5 Nonstandard Case

In the nonstandard case, the logarithm of the likelihood, except for a constant term, is given by

$$\log L = -(N/2)\{\log|\Sigma| + \mathrm{tr}[T(\Xi)\Sigma^{-1}]\} \quad , \tag{14}$$

where

$$T(\Xi) = (1/N)(X - A\Xi P)'(X - A\Xi P)$$

$$= W - P'\Xi'V - V'\Xi P + P'\Xi'U\Xi P \quad . \tag{15}$$

The maximum likelihood estimates are computed numerically by minimizing

$$F(\Xi,B,\Lambda,\Phi,\psi,\Theta) = \log|\Sigma| + \mathrm{tr}[T(\Xi)\Sigma^{-1}]$$

using a modification of the method of Fletcher and Powell (1963) (see Gruvaeus and Jöreskog, 1970). However, the minimization method is not applied directly to F but instead to

$$f(B,\Lambda,\Phi,\psi,\Theta) = \min_{\Xi} F(\Xi,B,\Lambda,\Phi,\psi,\Theta) \tag{16}$$

$$= F(\hat{\Xi}_{\Sigma},B,\Lambda,\Phi,\psi,\Theta) \quad ,$$

where $\hat{\Xi}_{\Sigma}$ minimizes F for given Σ. If Ξ is unconstrained,

$$\hat{\Xi}_{\Sigma} = U^{-1}V\Sigma^{-1}P'(P\Sigma^{-1}P')^{-1} \tag{17}$$

but this formula cannot be used if Ξ contains fixed and/or constrained elements. Nevertheless, $\hat{\Xi}_{\Sigma}$ can easily be evaluated since, for given Σ, F is quadratic in Ξ. The minimization of f takes into account the specification of fixed, free and constrained parameters. During the minimization, F is regarded as a function of the independent parameters $\theta' = (\theta_1, \theta_2, \ldots, \theta_m)$, say.

The minimization method is a rapidly converging iterative method that makes use of exact first-order derivatives and a symmetric matrix E of order $m \times m$. Initially E is obtained as the inverse of the information matrix $\mathcal{E}(\partial^2 f/\partial\theta\partial\theta')$ evaluated at the starting point. In subsequent iterations E is improved, using information built up about the function, so that ultimately E converges to an approximation of the inverse of $\partial^2 f/\partial\theta\partial\theta'$ at the minimum. When the minimum has been obtained, the inverse of $\mathcal{E}(\partial^2 f/\partial\theta\partial\theta')$ is computed again to give an estimate of the variance-covariance matrix of the estimators. This is used to obtain standard errors of the estimated parameters.

Three different estimates S, $\hat{T}$ and $\hat{\Sigma}$ of Σ are computed.

S is defined by (9) and is the maximum likelihood estimate under the condition that P is square and nonsingular and Σ is unconstrained.

$\hat{T}$ is the matrix $T(\hat{\Xi}_{\Sigma})$ evaluated at the minimum of F. If Σ is constrained, this estimate is not necessarily of the form (2).

$\hat{\Sigma}$ is the overall maximum likelihood estimate of Σ computed from (2) and evaluated at the minimum of F.

If Σ is unconstrained, $\hat{T}$ and $\hat{\Sigma}$ are identical. Otherwise, residual variances and covariances are defined as the elements of $\hat{T} - \hat{\Sigma}$.

Let H_0 be any specific hypothesis concerning the parametric structure of the general model and let H_1 be an alternative hypothesis. One can then test H_0 against H_1 by means of the likelihood ratio technique. Let F_0 be the minimum of F under H_0 and let F_1 be the minimum of F under H_1. Then $F_1 \leq F_0$ and minus two times the logarithm of the likelihood ratio becomes $N(F_0 - F_1)$. Under H_0 this is distributed, in large samples, as a χ^2 distribution with degrees of freedom equal to the difference in number of parameters estimated under H_1 and H_0.

2. The Program

In this section we describe briefly what the program does. Details about the input and output are given in sections 3 and 4 respectively.

2.1 What the Program Does

The input data may be the partitioned matrix (X/A), from which the matrices U, V and W are computed (see 1.3), or the matrices U, V and W, read in directly. In the standard case, other data matrices are P, C and D (see 1.1).

In the nonstandard case, the user can request an accurate or an approximate solution. If an accurate solution is requested, the iterations of the minimization method are continued until the minimum of the function is found, the convergence criterion being that the magnitude of all derivatives be less that .00005. The solution is then usually correct to three significant digits. If an approximate solution is requested, the iterations terminate when the decrease in function values is less than 5%. The approximate solution may be useless but the residuals and the value of χ^2 will usually give an indication of how reasonable the hypothesized model is. The option of an approximate solution has been included in the program for the purpose of saving computer time in exploratory studies where the primary purpose is to find a reasonable model. Once such a model has been found, an accurate solution may be computed. In the standard case, the user can test a sequence of hypotheses of the form $C\Xi D = 0$ for given U, V, W, P, C and D (see 1.1).

A variety of options for the printed output is available. In the nonstandard case, residuals may be printed, which are useful for judging the goodness of fit of the model to the data. These are the mea.. residuals defined as

$$V - U\hat{\Xi}P$$

and the residuals for Σ defined as

$$\hat{T} - \hat{\Sigma} \quad .$$

χ^2 is printed as an overall goodness of fit test statistic and standard errors for the estimated parameters may be requested. In the standard case, if testing the hypothesis $C\Xi D = 0$, the largest root, the sum of roots and the likelihood ratio, as described in section 1.4, will be printed. The large sample transformation of the likelihood ratio to a χ^2 is also printed.

2.2 How Fixed, Free and Constrained Parameters Are Specified

This section only applies to the nonstandard case (see 1.5). Since specifications for Ξ are slightly different from the specifications of the other parameter matrices, B , Λ , Φ , ψ and Θ , they will be described separately.

Specifications for B , Λ , Φ , ψ and Θ

The elements of the five matrices are ordered as follows. The matrices are assumed to be in the order B , Λ , Φ , ψ and Θ and within each matrix, the elements are ordered row-wise. The diagonal matrices ψ and Θ are treated as row vectors.

For each of the five parameter matrices, a pattern matrix is defined, with elements 0, 1, 2 and 3 depending on whether the corresponding element in the parameter matrix is fixed, free, constrained follower and constrained leader, respectively. A constrained parameter is called a constrained leader the first time it appears in the sequence. The parameters appearing later in the sequence and assumed to be equal to the constrained leader are called constrained followers.

The above technique defines uniquely the positions of the fixed, free and constrained leader parameters. It does not define, however, which followers go with which leader, if there is more than one leader. To do so one must also specify all the followers associated with a given leader. This is done by assigning to each leader and follower a five-digit number MRRCC, where M defines the matrix in which the constrained parameter appears ($M = 1$ for B , 2 for Λ , 3 for Φ , 4 for ψ and 5 for Θ), and RR and CC are the row and column position of the parameter in the matrix. For example,

10101 10201 10301 20403

defines $\beta_{11} = \beta_{21} = \beta_{31} = \lambda_{43}$ where β_{11} is the leader and β_{21} , β_{31} and λ_{43} are the followers. Such a string of numbers has to be provided for each leader.

Pattern matrices have to be provided for each matrix containing both fixed and free parameters and for each matrix containing constrained parameters. Patterns for parameter matrices whose elements are all fixed or all free are set up by the program.

We give a simple example to illustrate the above specifications. Suppose $\Lambda(2 \times 2) = I$, $\psi(2 \times 2) = 0$ and

$$B = \begin{bmatrix} \beta_{11} & 0 \\ \beta_{21} & 0 \\ 0 & \beta_{32} \\ 0 & \beta_{42} \end{bmatrix} \qquad \Phi = \begin{bmatrix} 1 & \\ \rho & 1 \end{bmatrix} \qquad \Theta = \begin{bmatrix} \theta_1 & 0 & 0 & 0 \\ 0 & \theta_2 & 0 & 0 \\ 0 & 0 & \theta_3 & 0 \\ 0 & 0 & 0 & \theta_4 \end{bmatrix}$$

with $\beta_{11} = \beta_{21}$, $\beta_{32} = \beta_{42}$, $\theta_1 = \theta_2$, $\theta_3 = \theta_4$. The pattern matrices for B , Φ and Θ are

$$P_B = \begin{bmatrix} 3 & 0 \\ 2 & 0 \\ 0 & 3 \\ 0 & 2 \end{bmatrix} \qquad P_\Phi = \begin{bmatrix} 0 & \\ 1 & 0 \end{bmatrix} \qquad P_\Theta = \begin{bmatrix} 3 & 2 & 3 & 2 \end{bmatrix}$$

and the specifications of leaders and followers are

10101	10201
10302	10402
50101	50102
50103	50104

In this model five independent parameters will be estimated.

Specifications for Ξ

The pattern for Ξ is defined in the same way as the patterns for B , Λ , Φ , Ψ , Θ . To specify what follower is associated with a given leader, a five-digit number MRRCC is assigned to each leader and follower as described above, with the one difference that M is always equal to 1 since we are dealing with only one matrix. A pattern matrix for Ξ must always be provided even if the elements of Ξ are all free or all fixed.

In addition to the above specifications for fixed, free and constrained parameters, start values have to be given for all parameters, except for those parameter matrices which are of standard form, i.e., $B = I$, $\Lambda = I$,

$\Phi = I$, $\Psi = 0$, $\Theta = 0$. Start values for Ξ are not read in if none of its entries are fixed. The start values define the fixed parameters and initial values for the minimization procedure for the other parameters, except for Ξ , whose initial values are set by the program to be equal to zero. Constrained parameters that are assumed to be equal must be given the same start values. Otherwise, initial values may be chosen arbitrarily but the closer they are to the final solution the less computer time it will take to reach this solution.

2.3 Limitations

The program is written in FORTRAN IV-G and has been tested on the 360/65 at Educational Testing Service. Double precision is used in floating point arithmetic throughout the program. With minor changes the program should run on any computer with a FORTRAN IV compiler. In computers with a single word length of 36 bits or more, single precision is probably sufficient.

Limitations as to the number of free and constrained parameters the program can handle and the storage requirements on the IBM 360/65 are given in the following table. The given storage requirements assume the program is overlayed.

Max. no. of variables	15
Max. no. of free and constrained parameters	60
Storage requirement (K = 1024 bytes)	132K

2.4 Availability

A copy of the program may be obtained, upon written request. The user must provide a tape on which the program will be loaded. The program will be written on the tape with 80 characters per record. The tape will be unlabeled. The user must specify whether he wants the tape blocked or unblocked, in EBCDIC or BCD mode, as well as the density, parity and track required. Test data will be at the end of the program. The test data are described in the Appendix. Anyone using the program for the first time should make sure that the test data run correctly.

2.5 Disclaimer

Although the program has been working satisfactorily for all data analyzed so far, no claim is made that it is free of error and no warranty is given as to the accuracy and functioning of the program.

3. Input Data

This section is divided into two parts. The first part will describe the input data for the standard case. The second part will describe the input data when the nonstandard case is considered (see 1.5).

In both cases, whenever a matrix or vector is read in it is preceded by a format card, containing at most 80 columns, beginning with a left parenthesis and ending with a right parenthesis. The format must specify floating point numbers for the input and parameter matrices, and fixed point numbers for the pattern matrices, consistent with the way in which the elements of the matrix are punched on the following cards. Users unfamiliar with FORTRAN are referred to a FORTRAN Manual, where format rules are given. Matrices are punched row-wise, each row beginning on a new card. For the symmetric matrices only the lower half of the matrix should be punched. The elements above the diagonal are automatically set by the program.

Part I: Standard Case

For each data to be analyzed, the input consists of the following:

1. Title card
2. Parameter card I
3. Rows of (X/A) (i.e. raw data to compute U , V , W)
4. Data matrices U , V , W
5. Data matrix P

6. Parameter card II

   or 1 blank card followed by the next data set (either for the standard case or the nonstandard case)

   or 1 blank card followed by a STOP card (see sec. 3.7)

7. Matrices C and D

8. Repeat steps 6 through 7

Sections 3.1 through 3.8 describe in general terms the function and setup of the above quantities.

3.1 Title Card

Whatever appears on this card will appear on the first page of the printed output. All 80 columns of the card are available to the user.

3.2 Parameter Card I

All quantities on this card except for the logical variables must be punched as integers right-adjusted within the field.

cols. 1-5: sample size (N), i.e., number of observations

cols. 6-10: number of variables (p) (≤ 15)

cols. 11-15: rank of A (g) (≤ 15)

cols. 16-20: number of rows in P (h) (≤ 15)

col. 41: logical indicator which determines whether U , V , W are computed from (X/A) or read in as input data

col. 41: = T , if rows of (X/A) are read in to compute U , V , W

col. 41: = F , if U , V , W are read in as input data col. 42: logical indicator which determines whether the data matrix P is equal to the identity or not

col. 42: = T , if $P = I$ (Note: only if $h = p$)

col. 42: = F , if $P \neq I$

col. 43: logical indicator which determines whether the same U , V , W as used in the previous data set will be used (never true for the first data set) in which case neither (X/A) nor U , V , W needs to be read in as input and col. 41 will be ignored

col. 43: = T , if new U , V , W are analyzed

col. 43: = F , if same U , V , W as previous data set are analyzed

3.3 Rows of (X/A)

Omit if col. 41 or col. 43 of parameter card I is false. Otherwise the partitioned matrix (X/A) is read in like any other input matrix. That is, it is preceded by a format card, read in row-wise where each row consists of a row of X immediately followed by a row of A , and a new card is started for each new row of (X/A) .

3.4 Data Matrices U , V , W

Omit if col. 41 of parameter card I is true or if col. 43 is false. Otherwise read in U , V , W respectively, each preceded by its format card. Since U and W are symmetric only their lower triangular parts, including the diagonal, are read in.

3.5 Data Matrix P

Omit if col. 42 of parameter card I is true. Otherwise read in P preceded by a format card.

3.6 Parameter Card II

All quantities on this card must be integers right-adjusted within the field.

cols. 1-5: number of rows in C (s) ($\leq$ 15)

cols. 6-10: number of columns in D (t) ($\leq$ 15)

3.7 Matrices C and D

Matrices C and D are read in consecutively, each preceded by its format card.

3.8 Stacked Data

The steps described in sections 3.6 and 3.7 can be repeated as many times as desired or they can be skipped altogether. The end of each standard data set must be followed by a blank card. This set can then be followed by a new data set, either for the standard case or the non-standard case. Any number of such data sets may be stacked together and analyzed in one run. Note: since the program looks for a blank card, any input cards for matrices with all zero rows should have the zero entries punched, i.e., do not use a blank card in lieu of all zero entries on an input card.

After the last set of data in the stack, there must be a card with the word STOP punched in columns 1-4.

Part II: Nonstandard Case

For each data to be analyzed, the input consists of the following:

1. Title card
2. Parameter card
3. Starting matrix Ξ
4. Specifications for Ξ
5. Rows of (X/A)
6. Data matrices U , V , W
7. Data matrix P
8. Pattern matrices for B , Λ , Φ , ψ and Θ
9. Equalities
10. Start values for B , Λ , Φ , ψ and Θ
11. New data set or STOP card

3.9 Title Card

Whatever appears on this card will appear on the first page of the printed output. All 80 columns of the card are available to the user.

3.10 Parameter Card

All quantities on this card except for the logical variables must be punched as integers right-adjusted within the field.

cols. 1-5: sample size (N), i.e., number of observations

cols. 6-10: number of variables (p) (≤ 15)

cols. 11-15: rank of A (g) (≤ 15)

cols. 16-20: number of rows in P (h) (≤ 15)

cols. 21-25: number of columns in B (q) (≤ 15)

cols. 26-30: number of columns in Λ (r) (≤ 15)

cols. 31-35: total estimated execution time in seconds for all stacked data (SEC). This should be a number slightly less than the time requested on the control cards so the program will have time to print and/or punch results up to that point. (Note: SEC should be read in for each nonstandard data set and should be the same for all such data sets in the stack.)

cols. 41-43: logical indicators (see below)

cols. 51-54: integer indicators (see below)

cols. 61-65: logical tape (disk) number of a scratch tape (disk) used in the program in the nonstandard case

Logical Indicators (cols. 41-43): The logical indicators control the input as described below.

Column 41 determines whether U , V , W are computed from (X/A) or read in as input data.

col. 41: = T , if rows of (X/A) are read in to compute U , V , W

col. 41: = F , if U , V , W are read in as input data

Column 42 determines whether the data matrix P is read in or set equal to the identity matrix by the program.

col. 42: = T , if P = I (note: only if h = p)

col. 42: = F , if P is read in as input data

Column 43 determines whether data matrix Ξ is read in or not. If no elements in Ξ are fixed, Ξ is not read in.

col. 43: $= T$, if Ξ is read in as input data

col. 43: $= F$, if Ξ is not read in

<u>Integer Indicators</u> (cols. 51-54).

<u>Column 51</u> determines the type of printed output wanted. This can be standard output (S), parameter specifications (R), the matrices T , Γ , Σ and residuals (C), and technical output (D). (See 4.2-4.5.)

col. 51: = 0 , for S

col. 51: = 1 , for S + R

col. 51: = 2 , for S + C

col. 51: = 3 , for S + R + C

col. 51: = 4 , for S + D

col. 51: = 5 , for S + R + D

col. 51: = 6 , for S + C + D

col. 51: = 7 , for S + R + C + D

<u>Column 52</u> determines whether the same U , V , W as used in the previous data set will be used (never true for the first data set) in which case neither (X/A) nor U , V , W need be read in as input (G). It also determines certain extra printed or punched output. This can be standard errors (F) and a punched solution (P). (See 4.6-4.7.)

col. 52: = 0 , if no extra output is wanted

col. 52: = 1 , for F

col. 52: = 2 , for P

col. 52: = 3 , for F + P

col. 52: = 4 , for G

col. 52: = 5 , for F + G

col. 52: = 6 , for P + G

col. 52: = 7 , for F + P + G

Column 53 determines whether an accurate or an approximate solution is required.

col. 53: = 0 , if an exact solution is required

col. 53: = 1 , if an approximate solution is required

Column 54 will be set to zero or left blank for ordinary purposes.

col. 54: = 0 , iterate and obtain all output requested through columns 51 and 52

col. 54: = 1 , no iterations (This may be used if one wants to test the goodness of fit of a solution which is completely specified.)

col. 54: = 2 , no standard output

3.11 Starting Matrix Ξ

Omit if col. 43 of the parameter card is false, otherwise read in matrix Ξ preceded by its format card. The program will set all free and constrained elements to zero, so only the fixed values read in are relevant.

3.12 Specifications for Ξ

A pattern matrix for Ξ is read in preceded by a format card (see 2.2). Note: this matrix is read in even if Ξ is not. The pattern matrix for Ξ is followed by "equality" cards, i.e., cards which determine which elements are followers and which are leaders (see 2.2). These "equality" cards are omitted if there are no elements equal to 2 or 3 in the pattern matrix for Ξ . Otherwise, starting in column 1 punch the five digit numbers MRRCC as described in section 2.2. For each new constrained leader start a new card. M is always equal to 1 on the "equality " cards specifying Ξ . The last entry on each "equality" card is a zero indicating more "equality" cards follow, or a two indicating it is the last one.

3.13 Rows of (X/A)

Omit if col. 41 of the parameter card is false or if col. 52 is greater than three. Otherwise, the partitioned matrix (X/A) is read in like any other input matrix. That is, it is preceded by a format card, is read in row-wise where each row consists of a row of X immediately followed by a row of A , and a new card is started for each new row.

3.14 Data Matrices U , V , W

Omit if col. 41 of the parameter card is true or if col. 52 is greater than three. Otherwise read in U , V , W respectively, each preceded by its format card. Since U and W are symmetric, only their lower triangular parts, including the diagonal, are read in.

3.15 Data Matrix P

Omit if col. 42 of the parameter card is true, otherwise read in matrix P preceded by a format card.

3.16 Pattern Matrices for B , Λ , Φ , ψ and Θ

These pattern matrices are preceded by a data card with entries in columns 1-5, the column defining the matrix in question, 1 for B , 2 for Λ , 3 for Φ , 4 for ψ and 5 for Θ .

cols. 1-5: CCCCC where C = 0 , if the matrix is fixed

C = 1 , if the matrix is free

C = 2 , if the matrix has mixed values

A pattern matrix should be provided only when C = 2 (see 2.2).

For example, if columns 1-5 are punched 20100, the matrix B contains mixed values, Λ is all fixed, Φ is all free, ψ and Θ are all fixed. In this case only a pattern matrix for B is read in.

The pattern matrix consists of a format card specifying an I -format and subsequent cards with the integer entries of the parameter matrix.

3.17 Equalities

Omit if the pattern matrices for B , Λ , Φ , ψ and Θ do not contain any elements 2 or 3. Otherwise starting in column 1 punch the five-digit numbers MRRCC as described in section 2.2. For each new constrained leader start a new card. The last entry on each "equality" card is a zero indicating more "equality" cards follow, or a six indicating it is the last one. The example in section 2.2 would then have the following equality cards:

10101102010
10302104020
50101501020
50103501046

3.18 Start Values for B , Λ , Φ , ψ and Θ

The start values are preceded by a data card with entries in columns 1-5, the column defining the matrix in question.

cols. 1-5: CCCCC where C = 0 , if the matrix is of standard form

C = 1 , otherwise (see 2.2)

This card is then followed by the necessary start values, for matrices with C = 1 , each matrix or vector with its own format card.

3.19 Stacked Data

In sections 3.9 through 3.18 we have described how each set of nonstandard data should be set up. Each such set of data can be followed by another data set, either for the standard case or the nonstandard case. (Note: a blank card does not indicate the end of a nonstandard data set--this is only true for the standard case (see 3.8).)

After the last set of data in the stack, there must be a card with the word STOP preceded in columns 1-4.

4. Printed and Punched Output

The output consists of a series of printed and punched tables as de-described in sections 4.1-4.7. Section 4.1 describes the output obtained when the standard case is considered. All subsequent sections deal with the various output options open to the user when considering the non-standard case.

4.1 Output for the Standard Case

The output for the standard case consists of the title with parameter listing, the matrices U , V , W , the matrices P , $\hat{\Xi}$, $\hat{\Sigma}$ and S (if $h = p$ $\hat{\Sigma}$ is not printed since $\Sigma = S$). Matrices C , D , S_e , S_h (see 1.4) and the three test statistics--the largest root, sum of roots, likelihood ratio and X^2 are printed when testing the hypothesis $C\Xi D = 0$.

The parameter listing gives the information supplied on parameter card I.

4.2 Standard Output (S) for the Nonstandard Case

The standard output is always obtained regardless of the value punched in columns 51 and 52 of the parameter card (see 3.10). The standard output consists of the title with parameter listing, the matrices U , V , W , P and S , the final solution and the result of the test of goodness of fit.

The parameter listing gives the information supplied on the parameter card.

The final solution consists of six matrices $\hat{\Xi}$, $\hat{B}$, $\hat{\Lambda}$, $\hat{\Phi}$, $\hat{\Psi}$ and $\hat{\Theta}$. All numbers are printed with three decimals.

The test of goodness of fit gives the value of χ^2 and the corresponding degrees of freedom. The probability level is also given. This is defined as the probability of getting a χ^2 value larger than that actually obtained, given that the hypothesized structure is true.

Just above the table giving the final solution, the following message is printed

'IND = X.'

Usually X is 0, but if, for some reason, it has not been possible to determine the final solution, X will be 1, 2, 3, 4 or 5. If IND is 1, 2 or 3, "serious problems" have been encountered and the minimization of the function cannot continue. One reason for this may be erroneous input data. Another reason may be that a point has been found where the matrix Σ is not positive definite. A third reason may be that insufficient arithmetic precision is used. If IND is 4, the number of iterations has exceeded 250. If IND is 5, the time limit SEC has been exceeded (see 3.10). If IND $\neq$ 0 , the solution obtained so far is automatically punched on cards. Each of the six matrices are preceded by a format card, so that they can immediately be used as initial estimates for a new run with the same data. Thus there is little loss of information when execution is terminated with IND $\neq$ 0 .

4.3 Parameter Specifications (R) for the Nonstandard Case

If column 51 of the parameter card is 1, 3, 5 or 7, a table of parameter specifications, containing the information provided by the pattern matrices (see 2.2), is printed. Six integer matrices are printed corresponding to Ξ , B , Λ , Φ , ψ and Θ . In each matrix an element is an integer equal to the index of the corresponding parameter in the sequence of independent parameters. The matrix Ξ has a sequence of independent parameters and the matrices B , Λ , Φ , ψ and Θ together form a second sequence of independent parameters. Elements corresponding to fixed parameters are 0 and elements corresponding to the same constrained parameter have the same value. Examples are given in the Appendix.

4.4 Matrices $\hat{T}$, $\hat{\Gamma}$, $\hat{\Sigma}$ and Residuals (C)

If column 51 of the parameter card is 2, 3, 6 or 7, the matrices $\hat{T}$ (see 1.5), $\hat{\Gamma} = \hat{\Lambda}\hat{\Phi}\hat{\Lambda}' + \hat{\psi}^2$, $\hat{\Sigma} = \hat{B}\hat{\Gamma}\hat{B}' + \hat{\Theta}^2$, the mean residuals = $\hat{V} - \hat{U}\hat{\Xi}\hat{P}$ and the residuals for $\Sigma = \hat{T} - \hat{\Sigma}$ are printed. The matrices $\hat{T}$, $\hat{\Gamma}$ and $\hat{\Sigma}$ are computed from the final solution. If the fit is good, $\hat{\Sigma}$ should agree with $\hat{T}$ and the residual matrix should be small. Elements of the residual matrix may suggest how the hypothesized structure should be modified to obtain a better fit. All five matrices are printed row-wise, each element with four decimals.

4.5 Technical Output (D)

If column 51 of the parameter card is 4, 5, 6 or 7, the technical output is printed. This consists of a series of tables that describe the behavior of the iterative procedure and give various measures of the accuracy of the final solution. Ordinary users will have little interest in these tables.

The first table of the technical output gives the initial estimates for Ξ, B, Λ, Φ, ψ and Θ.

The next two tables show the behavior of the iterative procedure under the steepest descent iterations and under the following iterations by the Fletcher and Powell method. For interpretation of these tables the reader is referred to Gruvaeus and Jöreskog (1970). If something goes wrong, so that IND is 1, 2 or 3 (see 4.2), these tables may contain valuable information.

4.6 Standard Errors (F)

If column 52 of the parameter card is 1, 3, 5 or 7, large sample approximations to the standard errors of the estimated parameters are printed. These are printed row-wise in matrix form and each number is printed with three decimals. The reader is referred to the paper by Jöreskog (1970a) for information about how the standard errors are obtained.

4.7 Punched Output (P)

If column 52 of the parameter card is 2, 3, 6 or 7, the final solution is punched on cards. These cards are punched in matrix form. Each matrix is preceded by a format card and each row of the matrix begins a new card.

References

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Appendix

We shall illustrate how input data are set up and what the printout looks like by means of two small sets of data. These data also serve as test data to be run when the program has been compiled on another computer. The various models and hypotheses have been chosen to illustrate the possibilities available in the program rather than the statistical problems involved.

Both sets of data are analyzed in one run. Pages A6-A9 show card by card how the input is punched. One line corresponds to one card. Pages A10-A36 show the corresponding printout obtained.

The first set of data is taken from Smith, Gnanadesikan and Hughes (1962) and consists of $N = 45$ observations on two covariables and $p = 11$ biochemical response measurements. The subjects were individuals classified into four weight groups. For further information about the subjects and the measurements, see the above reference and references therein.

The model is

$$\mathcal{E}(X_{45\times 11}) = A_{45\times 6}\Xi_{6\times 11} ,$$

i.e., $P_{11\times 11} = I$. The first four columns of A are used to classify individuals into weight groups and the last two consist of the measurements of the covariables. We do an ordinary MANOVA (standard case) and test the two hypotheses, H_1 that the first row of Ξ is zero and H_2 that rows 2, 3 and 4 of Ξ are all zero. These correspond to the hypotheses that the overall mean effect is zero and that there is no difference between weight groups, respectively. For both hypotheses we have $D_{11x11} = I$; for H_1 we have

$$C_{1x6} = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$$

and for H_2 we have

$$C_{3x6} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} .$$

For the analysis we use the raw data published in tables 2 and 3 of Smith et al. (1962).

The second set of data is taken from Potthoff and Roy (1964) and is used to illustrate the standard case with P rectangular $(h < p)$ and also the nonstandard case with both Ξ and Σ constrained. It consists of measurements on 11 girls and 16 boys at 4 different age levels.

Two analyses of these data are done. The input for the first analysis is the matrices U , V and W . Here we assume that Σ is unconstrained and that

$$\mathcal{E}(X_{27x4}) = A_{27x2}\Xi_{2x3}P_{3x4} \quad , \tag{A1}$$

where A is a matrix of zeros and ones with ones in column 1 for girls and ones in column 2 for boys and where

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 1 & 3 \\ 9 & 1 & 1 & 9 \end{bmatrix} .$$

The rows of ΞP represent two quadratic growth curves, one for girls and one for boys. We test two hypotheses H_1 and H_2 ; H_1 is that the coefficients ξ_{13} and ξ_{23} of the second-order terms are zero, i.e., that the growth curves are linear rather than quadratic; H_2 is that the two growth curves are the same, i.e., that the two rows of Ξ are identical. This amounts to choosing C and D as follows:

For H_1 : $C_{2x2} = I$ and $D_{3x1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

For H_2 = $C_{1x2} = (1 \quad -1)$

$D_{3x3} = I$.

The second analysis uses the same U , V and W as used previously and assumes that Σ has a quasi-Markov simplex structure with equal error variances (see Jöreskog, 1970, section 5.6). This may be represented as

$$\Sigma = \Lambda\Phi\Lambda' + \theta^2 I \quad ,$$

where

$$\Lambda = \begin{bmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \\ 0 & \lambda_{32} \\ 0 & \lambda_{42} \end{bmatrix} \quad ; \qquad \Phi = \begin{bmatrix} 1 & \\ \rho & 1 \end{bmatrix}$$

and θ is a scalar. In terms of (2) this is specified by choosing $B_{4x4} = I$, $\Lambda_{4x2} = \Lambda$, $\Phi_{2x2} = \Phi$, $\psi = 0$ and Θ constrained to have all diagonal elements equal. The model (A1) is the same but we now assume that

$\xi_{13} = \xi_{23} = 0$, $\xi_{11} = \xi_{21}$ and $\xi_{12} = \xi_{22}$. This analysis yields maximum likelihood estimates (accurate solution) of $\xi_{11} = \xi_{21}$, $\xi_{12} = \xi_{22}$, λ_{11} , λ_{21} , λ_{32} , λ_{42} , ρ , θ and an overall test of goodness of fit.

In the nonstandard case various time estimates are printed on the output. The time shown is the time taken to compute the solution that follows the time estimate. This time includes only the iterations and not the time for printing, except for the technical printout if requested.

References for Appendix

Jöreskog, K. G. Estimation and testing of simplex models. British Journal of Mathematical and Statistical Psychology, 1970, 23, in press.

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```
DATA FROM SMITH,GNANADESIKAN AND HUGHES (1962)
  45   11    6   11                                  TTT
(11F5.0)
  5.7 4.67 17.6  1.5 .104  1.5 1.88 5.15  8.4  7.5  .14
   1.   1.   0.   0. 205.  24.
  5.5 4.67 13.4 1.65 .245 1.32 2.24 5.75  4.5  7.1  .11
   1.   1.   0.   0. 160.  32.
  6.6  2.7 20.3   .9 .097  .89 1.28 4.35  1.2  2.3   .1
   1.   1.   0.   0. 480.  17.
  5.7 3.49 22.3 1.75 .174  1.5 2.24 7.55 2.75   4.  .12
   1.   1.   0.   0. 230.  30.
  5.6 3.49 20.5  1.4  .21 1.19   2.  8.5  3.3   2.  .12
   1.   1.   0.   0. 235.  30.
   6. 3.49 18.5  1.2 .275 1.03 1.8410.25   2.   2.  .12
   1.   1.   0.   0. 215.  27.
  5.3 4.84 12.1  1.9  .17 1.87  2.4 5.95  2.6 16.8  .14
   1.   1.   0.   0. 215.  25.
  5.4 4.84  12. 1.65 .164 1.68   3.  6.3 2.72 14.5  .14
   1.   1.   0.   0. 190.  30.
  5.4 4.84 10.1  2.3 .275 2.08 2.68 5.45  2.4   .9   .2
   1.   1.   0.   0. 190.  28.
  5.6 4.48 14.7 2.35  .21 2.55   3. 3.75   7.   2.  .21
   1.   1.   0.   0. 175.  24.
  5.6 4.48 14.8 2.35  .05 1.32 2.84  5.1   4.   .4  .12
   1.   1.   0.   0. 145.  26.
  5.6 4.48 14.4  2.5 .143 2.38 2.84 4.05   8.  3.8  .18
   1.   1.   0.   0. 155.  27.
  5.2 3.48 18.1  1.5 .153  1.2  2.6   9. 2.35 14.5  .13
   1.   0.   1.   0. 220.  31.
  5.2 3.48 19.7 1.65 .203 1.73 1.88  5.3 2.52 12.5   .2
   1.   0.   1.   0. 300.  23.
  5.6 3.48 16.9  1.4 .074 1.15 1.72 9.85 2.45   8.  .07
   1.   0.   1.   0. 305.  32.
  5.8 2.63 23.7 1.65 .155 1.58  1.6  3.6 3.75  4.9   .1
   1.   0.   1.   0. 275.  20.
   6. 2.63 19.2   .9 .155  .96  1.2 4.05  3.3   .2   .1
   1.   0.   1.   0. 405.  18.
  5.3 2.63  18.  1.6 .129 1.68   2.  4.4   3.  3.6  .18
   1.   0.   1.   0. 210.  23.
  5.4 4.46 14.8 2.45 .245 2.15 3.12 7.15 1.81  12.  .13
   1.   0.   1.   0. 170.  31.
  5.6 4.46 15.6 1.65 .422 1.42 2.56 7.25 1.92  5.2  .15
   1.   0.   1.   0. 235.  28.
  5.3  2.8 16.2 1.65 .063 1.62 2.04  5.3  3.9 10.2  .12
   1.   0.   1.   0. 185.  21.
  5.4  2.8 14.1 1.25 .042 1.62 1.84  3.1  4.1  8.5   .3
   1.   0.   1.   0. 255.  20.
  5.5  2.8 17.5 1.05  .03 1.56 1.48  2.4  2.1  9.6   .2
   1.   0.   1.   0. 265.  15.
  5.4 2.57 14.1  2.7 .194 2.77 2.56 4.25  2.6  6.9  .17
   1.   0.   1.   0. 305.  26.
  5.4 2.57 19.1  1.6 .139 1.59 1.88  5.8  2.3  4.7  .16
   1.   0.   1.   0. 440.  24.
  5.2 2.57 22.5  .85 .046 1.65  1.2 1.55  1.5  3.5  .21
   1.   0.   1.   0. 430.  16.
  5.5 1.26  17.   .7 .094  .97 1.24 4.55  2.9  1.9  .12
   1.   0.   0.   1. 350.  18.
  5.9 1.26 12.5   .8 .039   .8  .64 2.65  .72   .7  .13
   1.   0.   0.   1. 475.  10.
  5.6 2.52 21.5  1.8 .142 1.77  2.6  6.5 2.48  8.3  .17
```

```
  1.   0.   0.   1. 195.  33.
 5.6 2.52 22.2 1.05  .08 1.17 1.48 4.85  2.2  9.3  .14
  1.   0.   0.   1. 375.  25.
 5.3 2.52  13.  2.2 .215 1.85 3.84 8.75  2.4  13.  .11
  1.   0.   0.   1. 160.  35.
 5.6 3.24  13. 3.55 .166 3.18 3.48  5.2  3.5 18.3  .22
  1.   0.   0.   1. 240.  33.
 5.5 3.24 10.9  3.3 .111 2.79 3.04 4.75 2.52 10.5  .21
  1.   0.   0.   1. 205.  31.
 5.6 3.24  12. 3.65  .18  2.4   3. 5.85   3. 14.5  .21
  1.   0.   0.   1. 270.  34.
 5.4 1.56 22.8  .55 .069   1. 1.14 2.85  2.9  3.3  .15
  1.   0.   0.   1. 475.  16.
 5.3 1.56 16.5 2.05 .222 1.49  2.4 6.55  3.9  6.3  .11
  1.   0.   0.   1. 430.  31.
 5.2 1.56 18.4 1.05 .267 1.17 1.36  6.6   2.  4.9  .11
  1.   0.   0.   1. 490.  28.
 5.8 4.12 12.5  5.9 .093  3.8 3.84  2.9   3. 22.5  .24
  1.  -1.  -1.  -1. 105.  32.
 5.7 4.12  8.7 4.25 .147 3.62 5.32   3. 3.55 19.5   .2
  1.  -1.  -1.  -1. 115.  25.
 5.5 4.12  9.4 3.85 .217 3.36 5.52  3.4  5.2  1.3  .31
  1.  -1.  -1.  -1.  97.  28.
 5.4 2.14  15. 2.45 .418 2.38  2.4  5.4 1.81  20.  .17
  1.  -1.  -1.  -1. 325.  27.
 5.4 2.14 12.9  1.7 .323 1.74 2.48 4.45 1.88   1.  .15
  1.  -1.  -1.  -1. 310.  23.
 4.9 2.03 12.1  1.8 .205   2. 2.24  4.3  3.7   5.  .19
  1.  -1.  -1.  -1. 245.  25.
  5. 2.03 13.2 3.65 .348 1.95 2.12   5.  1.8   3.  .15
  1.  -1.  -1.  -1. 170.  26.
 4.9 2.03 11.5 2.25  .32 2.25 3.12  3.4  2.5  5.1  .18
  1.  -1.  -1.  -1. 220.  34.
   1   11
(80F1.0)
100000
(80F1.0)
1
01
001
0001
00001
000001
0000001
00000001
000000001
0000000001
00000000001
   3   11
(80F1.0)
01
001
0001
(80F1.0)
1
01
001
0001
00001
```

```
000001
0000001
00000001
000000001
0000000001
00000000001

DATA FROM POTTHOFF AND ROY (1964)     STANDARD CASE
   27     4     2     3              110      FFT
(16F5.0)
 .407
   0. .593
(8F10.0)
     8.630     9.056     9.407     9.815
    13.556    14.111    15.241    16.278
(8F10.0)
   497.889
   517.120   541.176
   551.518   574.731   615.176
   582.759   608.843   649.102   688.194
(10F3.0)
  1  1  1  1
 -3 -1  1  3
  9  1  1  9
    2     1
(10F1.0)
10
01
(10F1.0)
0
0
1
    1     3
(10F5.0)
    1    -1
(10F1.0)
1
01
001

DATA FROM POTTHOFF AND ROY(1964) NON-STANDARD CASE,BOTH XI AND SIGMA CONSTRAINED
   27     4     2     3     4     2  110      FFT         7500            4
(10F1.0)
000
000
(80I1)
330
220
10101102010
10102102022
(10F3.0)
  1  1  1  1
 -3 -1  1  3
  9  1  1  9
02202
(80I1)
10
10
01
01
```

```
(8011)
0
10
(8011)
3222
501015010250103501046
01101
(16F5.0)
   2.    0.
   2.    0.
   0.    2.
   0.    2.
(16F5.0)
   1.
   .8    1.
(80F1.0)
1111
STOP
```

ANALYSIS OF COVARIANCE STRUCTURES

DATA FROM SMITH,GNANADESIKAN AND HUGHES (1962)

N= 45

P= 11

G= 6

H= 11

LOGICAL VARIABLES (COLS.41-43) : TTT

U=(1/N)A'A

	1	2	3	4	5	6
1	1.000					
2	0.089	0.444				
3	0.133	0.178	0.489			
4	0.067	0.178	0.178	0.422		
5	263.267	22.400	53.622	46.178	80777.973	
6	25.822	2.222	2.400	1.644	6471.800	700.267

V=(1/N)A'X

	1	2	3	4	5	6	7	8	9	10
1	5.498	3.134	15.896	1.998	0.174	1.815	2.382	5.247	3.076	7.467
2	0.564	0.616	2.120	-0.098	0.001	-0.040	0.027	0.896	0.565	-0.313
3	0.749	0.458	3.427	-0.088	-0.000	0.035	0.014	0.914	0.315	0.598
4	0.398	0.039	1.878	-0.114	-0.011	-0.056	-0.063	0.606	0.113	0.302
5	1452.033	749.315	4398.673	456.704	43.996	434.364	551.781	1351.434	742.113	1776.747
6	141.544	83.131	403.142	54.788	4.735	48.548	64.884	142.191	80.309	206.367

V=(1/N)A'X

	11
1	0.160
2	0.002
3	0.014
4	0.002
5	40.304
6	4.131

W=(1/N)X'X

	1	2	3	4	5	6	7	8	9	10
1	30.312									
2	17.287	10.924								
3	87.654	48.571	267.732							
4	10.965	6.639	29.213	5.107						
5	0.948	0.551	2.664	0.368	0.039					
6	9.948	5.930	27.133	4.309	0.327	3.810				
7	13.060	7.953	35.361	5.583	0.438	4.899	6.623			
8	28.849	16.866	84.389	10.204	0.975	9.088	12.533	31.370		
9	16.930	10.339	48.147	6.433	0.515	5.859	7.732	15.673	11.837	
10	40.948	24.858	112.204	18.064	1.297	15.740	20.224	40.172	22.234	89.877
11	0.875	0.509	2.457	0.346	0.028	0.315	0.404	0.786	0.508	1.257

W=(1/N)X'X

	11
11	0.028

P

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000

11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

P

	11
1	0.0
2	0.0
3	0.0
4	0.0
5	0.0
6	0.0
7	0.0
8	0.0
9	0.0
10	0.0
11	1.000

XI

	1	2	3	4	5	6	7	8	9	10
1	5.790	3.667	10.938	2.312	-0.157	2.448	2.739	-3.142	6.147	3.161
2	0.195	0.895	1.109	-0.549	0.003	-0.429	-0.369	0.915	0.741	-3.124
3	-0.074	0.205	1.883	-0.321	-0.018	-0.145	-0.247	0.651	-0.327	0.944
4	0.013	-0.523	-0.419	0.091	-0.068	0.045	0.088	-0.311	0.071	1.391
5	0.000	-0.005	0.017	-0.005	0.000	-0.003	-0.005	0.005	-0.007	-0.013
6	-0.013	0.026	0.007	0.039	0.010	0.012	0.042	0.269	-0.050	0.303

XI

	11
1	0.286
2	-0.030
3	-0.004
4	0.009
5	-0.000
6	-0.002

SIGMA=S

	1	2	3	4	5	6	7	8	9	10
1	0.066									
2	0.035	0.279								
3	0.087	-0.502	8.585							
4	0.064	0.150	-0.523	0.425						
5	-0.003	-0.003	-0.020	-0.011	0.005					
6	0.023	0.119	-0.455	0.253	-0.006	0.249				
7	0.037	0.149	-0.563	0.161	-0.004	0.169	0.292			
8	0.025	-0.199	0.677	-0.397	0.027	-0.412	-0.210	1.776		
9	-0.025	0.058	0.208	0.074	-0.016	0.152	0.107	-0.605	1.709	
10	0.165	1.049	-1.622	1.088	-0.091	0.921	0.444	-0.645	-0.760	25.133
11	-0.001	0.007	-0.030	0.010	-0.000	0.013	0.009	-0.032	0.012	0.021

SIGMA=S

	11
11	0.002

TEST OF THE HYPOTHESIS C(X)D=0

C

	1	2	3	4	5	6
1	1.000	0.0	0.0	0.0	0.0	0.0

D

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

D

	11
1	0.0
2	0.0
3	0.0
4	0.0
5	0.0
6	0.0
7	0.0
8	0.0
9	0.0
10	0.0
11	1.000

S(E)

	1	2	3	4	5	6	7	8	9	10
1	0.066									
2	0.035	0.279								
3	0.087	-0.502	8.585							
4	0.064	0.150	-0.523	0.425						
5	-0.003	-0.003	-0.020	-0.011	0.005					
6	0.023	0.118	-0.455	0.253	-0.006	0.240				
7	0.037	0.149	-0.563	0.161	-0.004	0.169	0.292			
8	0.025	-0.190	0.677	-0.397	0.027	-0.412	-0.210	1.726		
9	-0.025	0.058	0.208	0.074	-0.016	0.152	0.107	-0.605	1.709	
10	0.165	1.048	-1.622	1.088	-0.091	0.921	0.4[illegible]	-0.645	-0.760	25.133
11	-0.001	0.007	-0.030	0.010	-0.000	0.013	0.00[illegible]	-0.032	0.012	0.021

S(E)

	11
11	0.002

S(H)

	1	2	3	4	5	6	7	8	9	10
1	0.504									
2	0.319	0.202								
3	0.952	0.603	1.798							

4	0.201	0.127	0.380	0.080						
5	-0.014	-0.009	-0.026	-0.005	0.000					
6	0.213	0.135	0.402	0.085	-0.006	0.090				
7	0.238	0.151	0.450	0.095	-0.006	0.101	0.113			
8	-0.273	-0.173	-0.517	-0.109	0.007	-0.116	-0.129	0.148		
9	0.535	0.339	1.011	0.214	-0.014	0.226	0.253	-0.290	0.568	
10	0.275	0.174	0.520	0.110	-0.007	0.116	0.130	-0.149	0.292	0.150
11	0.025	0.016	0.047	0.010	-0.001	0.011	0.012	-0.013	0.026	0.014

S(H)

	11
11	0.001

LARGEST ROOT= 0.11054140 02
SUM OF ROOTS= 0.1105414D 02
LIKELIHOOD RATIO= 0.8295905D-01
CHISQUARE WITH 11 DEGREES OF FREEDOM IS 83.3952

TEST OF THE HYPOTHESIS C(X)D=0

C

	1	2	3	4	5	6
1	0.0	1.000	0.0	0.0	0.0	0.0
2	0.0	0.0	1.000	0.0	0.0	0.0
3	0.0	0.0	0.0	1.000	0.0	0.0

D

	1	2	3	4	5	6	7	8	9	10
1	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.000
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

D

	11
1	0.0
2	0.0
3	0.0
4	0.0
5	0.0
6	0.0
7	0.0
8	0.0
9	0.0
10	0.0
11	1.000

S(E)

	1	2	3	4	5	6	7	8	9	10
1	0.066									
2	0.035	0.279								
3	0.087	-0.502	8.585							
4	0.064	0.150	-0.523	0.425						
5	-0.003	-0.003	-0.020	-0.011	0.005					
6	0.023	0.118	-0.455	0.253	-0.006	0.240				
7	0.037	0.149	-0.563	0.161	-0.004	0.169	0.292			
8	0.025	-0.190	0.677	-0.397	0.027	-0.412	-0.210	1.726		
9	-0.025	0.058	0.208	0.074	-0.016	0.152	0.107	-0.605	1.709	
10	0.165	1.048	-1.622	1.088	-0.091	0.921	0.444	-0.645	-0.760	25.133
11	-0.001	0.007	-0.030	0.010	-0.000	0.013	0.009	-0.032	0.012	0.021

S(E)

	11
11	0.002

S(H)

	1	2	3	4	5	6	7	8	9	10
1	0.015									

2	0.053	0.321								
3	0.075	0.669	2.426							
4	-0.039	-0.233	-0.666	0.211						
5	-0.002	-0.003	-0.037	0.011	0.002					
6	-0.031	-0.165	-0.431	0.145	0.007	0.102				
7	-0.026	-0.161	-0.469	0.146	0.007	0.099	0.101			
8	0.061	0.404	1.168	-0.358	-0.016	-0.243	-0.249	0.618		
9	0.057	0.193	0.250	-0.142	-0.006	-0.115	-0.092	0.218	0.219	
10	-0.193	-0.856	-0.920	0.503	-0.006	0.401	0.336	-0.832	-0.735	3.111
11	-0.002	-0.010	-0.022	0.008	0.000	0.006	0.006	-0.014	-0.007	0.029

S(H)

	11
11	0.000

LARGEST ROOT= 0.5556551D 01
SUM OF ROOTS= 0.6441797D 01
LIKELIHOOD RATIO= 0.7341101D-01
CHISQUARE WITH 33 DEGREES OF FREEDOM IS 90.1030

```
ANALYSIS OF COVARIANCE STRUCTURES
DATA FROM POTTHOFF AND ROY (1964)     STANDARD CASE
N=    27
P=     4
G=     2
H=     3
LOGICAL VARIABLES (COLS.41-43) :  FFT
```

U=(1/N)A'A

	1	2
1	0.407	
2	0.0	0.593

V=(1/N)A'X

	1	2	3	4
1	8.630	9.056	9.407	9.815
2	13.556	14.111	15.241	16.278

W=(1/N)X'X

	1	2	3	4
1	497.889			
2	517.120	541.176		
3	551.518	574.731	615.176	
4	582.759	608.843	649.102	688.194

P

	1	2	3	4
1	1.000	1.000	1.000	1.000
2	-3.000	-1.000	1.000	3.000
3	9.000	1.000	1.000	9.000

XI

	1	2	3
1	22.704	0.479	-0.003
2	24.631	0.788	0.050

S

	1	2	3	4
1	5.008			
2	2.519	3.890		
3	3.643	2.745	6.035	
4	2.527	3.103	3.878	4.665

SIGMA

	1	2	3	4
1	5.021			
2	2.511	3.896		
3	[illegible].663	2.731	6.069	
4	2.527	3.102	3.879	4.665

```
TEST OF THE HYPOTHESIS C(XI)D=0

          C

                  1         2
        1         1.000     0.0
        2         0.0       1.000

          D

                  1
        1         0.0
        2         0.0
        3         1.000

          S(E)

                  1
        1         0.015

          S(H)

                  1
        1         0.001

LARGEST ROOT=  0.99323620-01
SUM OF ROOTS=  0.99323620-01
LIKELIHOOD RATIO=  0.90965020 00
CHISQUARE WITH    2 DEGREES OF FREEDOM IS          2.2727
```

TEST OF THE HYPOTHESIS C(XI)D=0

C

	1	2
1	1.000	-1.000

D

	1	2	3
1	1.000	0.0	0.0
2	0.0	1.000	0.0
3	0.0	0.0	1.000

S(E)

	1	2	3
1	3.845		
2	0.137	0.109	
3	-0.088	-0.012	0.015

S(H)

	1	2	3
1	0.896		
2	0.144	0.023	
3	0.025	0.004	0.001

LARGEST ROOT= 0.63029720 00
SUM OF ROOTS= 0.63029720 00
LIKELIHOOD RATIO= 0.61338510 00
CHISQUARE WITH 3 DEGREES OF FREEDOM IS 10.9972

ANALYSIS OF COVARIANCE STRUCTURES

DATA FROM POTTHOFF AND ROY(1964) NON-STANDARD CASE,BOTH XI AND SIGMA CONSTRAINED

N= 27

P= 4

G= 2

H= 3

Q= 4

R= 2

LOGICAL INDICATORS (COLS.41-43) : FFT

INDICATORS (COLS.51-54) : 7500

ESTIMATED TIME IN SECONDS= 110.

U=(1/N)A'A

	1	2
1	0.407	
2	0.0	0.593

V=(1/N)A'X

	1	2	3	4
1	8.630	9.056	9.407	9.815
2	13.556	14.111	15.241	16.278

W=(1/N)X'X

	1	2	3	4
1	497.889			
2	517.120	541.176		
3	551.518	574.731	615.176	
4	582.759	608.843	649.102	688.194

P

	1	2	3	4
1	1.000	1.000	1.000	1.000
2	-3.000	-1.000	1.000	3.000
3	9.000	1.000	1.000	9.000

S = W-V'(U**-1)V

	1	2	3	4
1	5.008			
2	2.519	3.890		
3	3.643	2.745	6.035	
4	2.527	3.103	3.878	4.665

PARAMETER SPECIFICATIONS

XI

```
1  2  0
1  2  0
```

BETA

```
0  0  0  0
0  0  0  0
0  0  0  0
0  0  0  0
```

LAMBDA

```
1  0
2  0
0  3
0  4
```

PHI

```
0
5  0
```

PSI

```
0  0  0  0
```

THETA

```
6  6  6  6
```

INITIAL SOLUTION

XI

	1	2	3
1	0.0	0.0	0.0
2	0.0	0.0	0.0

BETA

	1	2	3	4
1	1.000	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0
3	0.0	0.0	1.000	0.0
4	0.0	0.0	0.0	1.000

LAMBDA

	1	2
1	2.000	0.0
2	2.000	0.0
3	0.0	2.000
4	0.0	2.000

PHI

	1	2
1	1.000	
2	0.800	1.000

PSI

	1	2	3	4
1	0.0	0.0	0.0	0.0

THETA

	1	2	3	4
1	1.000	1.000	1.000	1.000

BEHAVIOR UNDER STEEPEST DESCENT ITERATIONS

ITER	TRY	ABSCISSA	SLOPE	FUNCTION
1	0	0.0	-0.31502167D 01	0.13616159D 01
	1	0.10000000D 00	-0.17469813D 01	0.11214663D 01
	2	0.22449668D 00	-0.61171792D 00	0.98006172D 00
	3	0.34861600D 00	0.11452977D 00	0.95239137D 00
2	0	0.0	-0.79462110D 00	0.95239137D 00
	1	0.34861600D 00	0.27171471D 02	0.21198742D 01
	2	0.19199531D 00	0.10960096D 01	0.91843175D 00
	3	0.12578872D 00	-0.48286142D-01	0.88912964D 00
3	0	0.0	-0.49788987D 00	0.88912964D 00
	1	0.12578872D 00	-0.60810765D-01	0.85616913D 00
	2	0.15022973D 00	0.30723980D-02	0.85547464D 00
4	0	0.0	-0.65404741D 00	0.85547464D 00
	1	0.15022973D 00	0.16576672D 01	0.87854984D 00
	2	0.76436948D-01	0.95550460D-02	0.82722700D 00

BEHAVIOR UNDER FLEPOW ITERATIONS

ITER	TRY	ABSCISSA	SLOPE	FUNCTION
1	0	0.0	-0.27831721D 00	0.82722700D 00
	1	0.10000000D 00	-0.20048700D 00	0.80333422D 00
	2	0.40198130D 00	0.15827383D-02	0.77450242D 00
2	0	0.0	-0.12050417D 00	0.77450242D 00
THE MATRIX IS NOT POSITIVE DEFINITE		4	-3.94206063	3.92787721
	1	0.40198130D 00		
	2	0.20099065D 00	0.83725496D 01	0.12258308D 01
	[illegible]	0.57237105D-01	0.54566738D 00	0.78460413D [illegible]
	4	0.13749608D-01	0.26084690D-02	0.77367077D [illegible]
3	0	0.0	-0.52224024D-02	0.77367077D 00
	1	0.13749608D-01	0.90009837D-02	0.77369664D 00
	2	0.50591060D-02	-0.97209537D-08	0.77365755D 00
4	0	0.0	-0.32548881D-03	0.77365755D 00
	1	0.50591060D-02	0.68607513D-03	0.77365847D 00
	2	0.16278579D-02	-0.36988761D-06	0.77365729D 00
5	0	0.0	-0.59229782D-04	0.77365729D 00
	1	0.16278579D-02	0.26723782D-02	0.77365941D 00
	2	0.35361391D-04	0.11505241D-09	0.77365729D 00

TIME= 2.50

MAXIMUM LIKELIHOOD SOLUTION

IND= 0

XI

	1	2	3
1	23.949	0.654	0.0
2	23.949	0.654	0.0

BETA

	1	2	3	4
1	1.000	0.0	0.0	0.0
2	0.0	1.000	0.0	0.0
3	0.0	0.0	1.000	0.0
4	0.0	0.0	0.0	1.000

LAMBDA

	1	2
1	1.921	0.0
2	1.683	0.0
3	0.0	2.454
4	0.0	2.393

PHI

	1	2
1	1.000	
2	0.949	1.000

PSI

	1	2	3	4
1	0.0	0.0	0.0	0.0

THETA

	1	2	3	4
1	1.319	1.319	1.319	1.319

T

	1	2	3	4
1	5.710			
2	3.111	4.482		
3	4.686	3.705	7.654	
4	3.896	4.323	5.970	7.382

GAMMA = (LAMBDA)(PHI)(LAMBDA)'+(PSI)**2

	1	2	3	4
1	3.690			
2	3.233	2.832		
3	4.473	3.919	6.023	
4	4.362	3.822	5.873	5.727

SIGMA = (BETA)(GAMMA)(BETA)'+(THETA)**2

	1	2	3	4
1	5.429			
2	3.233	4.571		
3	4.473	3.919	7.762	
4	4.362	3.822	5.873	7.466

MEAN RESIDUALS = V-(U)(XT)(P)

	1	2	3	4
1	-0.318	-0.425	-0.607	-0.731
2	0.518	0.297	0.651	0.912

RESIDUALS FOR SIGMA = T-SIGMA

	1	2	3	4
1	0.281			
2	-0.122	-0.089		
3	0.213	-0.214	-0.108	
4	-0.466	0.502	0.096	-0.084

TEST OF GOODNESS OF FIT

CHISQUARE WITH 10 DEGREES OF FREEDOM IS 20.1151

PROBABILITY LEVEL IS 0.028

STANDARD ERRORS

XI

	1	2	3
1	0.404	0.065	0.0
2	0.404	0.065	0.0

BETA

	1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0

LAMBDA

	1	2
1	0.379	0.0
2	0.357	0.0
3	0.0	0.428
4	0.0	0.421

PHI

	1	2
1	0.0	
2	0.080	0.0

PSI

	1	2	3	4
1	0.0	0.0	0.0	0.0

THETA

	1	2	3	4
1	0.127	0.127	0.127	0.127